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Week 07 ▪ 041-325

Zoom Class 14/04/2020.

Mathematics Blog 2

Total differentiatiale Coefficient

let $u = f(x,y)$ where $x = \phi(t)$, $y = \psi(t)$
i.e., x is a function of t and
y is also a function of t.

then $$\delta u = \frac{f(x+\delta x, y+\delta y) - f(x, y+\delta y)}{\delta x} \cdot \delta x$$
$$+ \frac{f(x, y+\delta y) - f(x,y)}{\delta y} \cdot \delta y \quad \text{--- } (1)$$

$\hookrightarrow$ {using the total differentiation formula
$$du = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} \cdot dy \}$$

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Differentiating Eqn (1) w.r.t 't' we get

$$\frac{\delta u}{\delta t} = \frac{f(x+\delta x, y+\delta y) - f(x, y+\delta y)}{\delta x} \cdot \frac{\delta x}{\delta t}$$
$$+ \frac{f(x, y+\delta y) - f(x,y)}{\delta y} \cdot \frac{\delta y}{\delta t} \quad \text{--- } (2)$$

Now in Eqn (2) as $\delta t \to 0$ then $\delta x \to 0$ and $\delta y \to 0$

$$\Rightarrow \boxed{\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}}$$

Similarly if $u = f(x, y, z)$ & $x = \phi_1(t), y = \phi_2(t), z = \phi_3(t)$

$$\Rightarrow \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}.$$

Similar sequence is valid for generalization upto n variables i.e.,

$$u = f(x_1, x_2, x_3, \dots, x_n)$$

Second Proof of Euler's theorem using total diff$^{\underline{n}}$

If u is a homogeneous function of n^{th} order in x, y, z such that

$$u = x^n f\left(\frac{y}{x}, \frac{z}{x}\right) \longrightarrow (1)$$

then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$

Proof: let $\frac{y}{x} = Y$ and $\frac{z}{x} = Z$

$$\Rightarrow \frac{\partial Y}{\partial x} = -\frac{y}{x^2} \text{ and } \frac{\partial Y}{\partial y} = \frac{1}{x}, \text{ and } \frac{\partial Y}{\partial z} = 0$$

&

$$\frac{\partial Z}{\partial x} = \frac{-z}{x^2} \text{ and } \frac{\partial Z}{\partial y} = 0, \quad \frac{\partial Z}{\partial z} = \frac{1}{x}$$

$(1) \Rightarrow u = x^n f(Y, Z) \longrightarrow (2)$

$$\Rightarrow \frac{\partial u}{\partial x} = nx^{n-1} f(Y, Z) + x^n \left\{\frac{\partial f}{\partial Y} \cdot \frac{\partial Y}{\partial x} + \frac{\partial f}{\partial Z} \cdot \frac{\partial Z}{\partial x}\right\}$$

$\longrightarrow$ {Differentiating in the form of $u \cdot v$}

$$\frac{\partial u}{\partial x} = nx^{n-1} f(Y,Z) + x^n \left\{ \frac{\partial f}{\partial Y} \cdot \left(\frac{-y}{x^2} \right) + \frac{\partial f}{\partial Z} \left(\frac{-z}{x^2} \right) \right\}$$

(substituting above obtained values)

$$= nx^{n-1} f(Y,Z) - x^{n-2} \left\{ y \frac{\partial f}{\partial Y} + z \frac{\partial f}{\partial Z} \right\} \quad \text{--- (3)}$$

(taking $\frac{1}{x^2}$ common)

Sly
$$\frac{\partial u}{\partial y} = x^n \left\{ \frac{\partial f}{\partial Y} \cdot \frac{\partial Y}{\partial y} + \frac{\partial f}{\partial Z} \cdot \frac{\partial Z}{\partial y} \right\}$$

$$= x^n \left\{ \frac{\partial f(Y,Z)}{\partial Y} \cdot \frac{1}{x} + \frac{\partial f(Y,Z)}{\partial Z} \cdot 0 \right\}$$

$$= x^{n-1} \left\{ \frac{\partial f(Y,Z)}{\partial Y} \right\} \quad \text{--- (4)}$$

$$\frac{\partial u}{\partial Z} = x^n \left\{ \frac{\partial f(Y,Z)}{\partial Y} \cdot \frac{\partial Y}{\partial z} + \frac{\partial f(Y,Z)}{\partial Z} \cdot \frac{\partial Z}{\partial z} \right\}$$

$$= x^n \left\{ \frac{\partial f(Y,Z)}{\partial Y} \cdot 0 + \frac{\partial f}{\partial Z} \cdot \frac{1}{x} \right\}$$

$$= x^{n-1} \frac{\partial f}{\partial Z} \quad \text{--- (5)}$$

Hence from (3), (4), (5) we get

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial Z} = x n x^{n-1} f - \cancel{x^{n-2} \left\{ y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial Z} \right\}}$$

$$+ \cancel{y x^{n-1} \frac{\partial f}{\partial y}} + \cancel{z x^{n-1} \frac{\partial f}{\partial Z}}$$

$$= n x^n f(Y,Z) = nu$$

Proved

1^{st} differential Co-eff. of an implicit function

$$\boxed{\frac{dy}{dx} = -\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial y}}$$

If $u = f(x,y)$ and when the relationship b/w x and y is in form of an implicit function.

2^{nd} Differential Co-eff of implicit function

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$$\boxed{\frac{d^2y}{dx^2} = -\frac{q^2r - 2pqs + p^2t}{q^3}}$$

where $p = \frac{\partial f}{\partial x}$, $q = \frac{\partial f}{\partial y}$, $r = \frac{\partial^2 f}{\partial x^2}$,

$s = \frac{\partial^2 f}{\partial x \partial y}$, $t = \frac{\partial^2 f}{\partial y^2}$

Also $u = f(x,y)$ and relationship b/w x and y is an implicit function.